Real and p-Adic Aspects of Quantization of Tachyons

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Abstract. A simplified model of tachyon matter in classical and quantum mechanics is constructed. p-Adic path integral quantization of the model is considered. Recent results in using p-adic analysis, as well as perspectives of an adelic generalization, in the investigation of tachyons are briefly discussed. In particular, the perturbative approach in path integral quantization is proposed.

1 Introduction

An increasing number of researchers are testing again an (almost twenty year) old idea that p-adics and p-adic string theory [1] can be useful in attempts to understand ordinary string theory, D-brane solutions and various aspects of tachyons [2, 3]. Possible cosmological implications [4] are also an interesting matter.

p-Adic strings have many properties similar to ordinary strings, but p-adic ones are much simpler. For example, one can find an exact action for this theory, as well as for practically all variations of p-adic string theory. In addition, it turns out that p-adic string could be a very useful model for testing Sen's conjecture on tachyon condensation in open string field theory (see e.g. Ref. 5). Corresponding p-adic classical solution in string field theory can be explicitly found.

An important fact should be also considered, that is, p-adic string field theory in the $p \to 1$ limit reduces to the tachyon effective action [6]. It might be the case that results for p-adic strings are applicable to boundary string field theory. The next reason for further investigation is similarity between p-adic D-branes (without strong coupling problems) and the results found in vacuum cubic string filed theory. It is interesting that the effective energy-momentum tensor is equivalent to that of nonrotating dust [7].

Generally speaking, after almost twenty years of "discovering" p-adic strings,

our understanding of physics on p-adic spaces is still very poor. From many point of view, including pedagogical one, it is very useful to understand (quantum) mechanical analogies of new, unfamiliar objects, including tachyons. Considering "p-adic tachyons" we stress results in foundation of p-adic quantum mechanics, p-adic quantum cosmology and their connection with standard theory on real numbers (and corresponding spaces: Minkowsky, Riemman ...) and adelic quantum theory.

It has been noted that path integrals are extremely useful in this approach. Following S. Kar's [8] idea on the possibility of the examination of zero dimensional theory of the field theory of (real) tachyon matter, we extend this idea to the p-adic case. In addition, we note possibilities for adelic quantum treating of tachyon matter.

After some mathematical background in Section 2, we present p-adic path integrals in Section 3. Section 4 is devoted to p-adic strings and tachyons. Simple quantum mechanical analog of p-adic tachyons is considered in Section 5. The paper is ended by a short conclusion and suggestion for further research.

2 p-Adic Numbers and Related Analysis

Let us recall that all numerical experimental results belong to the field of rational numbers Q. The completion of this field with respect to the standard norm $|\ |_\infty$ (absolute value) leads to the field of real numbers $R \equiv Q_\infty$. Completion of Q with respect to the p-adic norms yields the fields of p-adic numbers Q_p (p is a prime number). Each non-trivial norm (valuation) on Q, due to the Ostrowski theorem, is equivalent either to a p-adic norm $|\ |_p$ or to the absolute value function.

Any p-adic number $x \in Q_p$ can be presented as an expansion [9]

$$x = x^\nu (x_0 + x_1 p + x_2 p^2 + \cdots), \quad \nu \in Z, \tag{1}$$

where $x_i = 0, 1, ..., p-1$. p-Adic norm of any term $x_i p^{\nu+i}$ in (1) is $p^{-(\nu+i)}$. The p-adic norm is the nonarchimedean (ultrametric) one. There are a lot of exotic features of p-adic spaces. For example, any point of a disc $B_\nu(a) = \{x \in Q_p : |x-a|_p \leq p^\nu\}$ can be treated as its center. It also leads to the total disconnectedness of p-adic spaces.

For the foundation of the path integral approach on p-adic spaces it is important to stress that no natural ordering on Q_p exists. However, one can define a linear order as follows: $x < y$ if $|x|_p < |y|_p$, or when $|x|_p = |y|_p$, when there is an index $m \geq 0$ such that the following is satisfied: $x_0 = y_0, x_1 = y_1, ..., x_{m-1} = y_{m-1}, x_m < y_m$ [10]. Generally speaking, there are two analysis over Q_p. One of them is connected with map $\phi : Q_p \to Q_p$, and the second one is related to the map $\psi : Q_p \to C$.

In the case of p-adic valued function, derivatives of $\phi(x)$ are defined as in the real case, using p-adic norm instead of the absolute value. p-Adic valued

definite integrals are defined for analytic functions

$$\phi(t) = \sum_{n=0}^{\infty} \phi_n t^n, \qquad \phi_n, t \in Q_p, \tag{2}$$

as follows:

$$\int_a^b \phi(t)dt = \sum_{n=0}^{\infty} \frac{\phi_n}{n+1}(b^{n+1} - a^{n+1}). \tag{3}$$

In the case of mapping $Q_p \to C$, standard derivatives are not possible, and several different types of pseudodifferential operators have been introduced [9, 11]. Contrary, there is a well defined integral with the Haar measure. Of special importance is Gauss integral

$$\int_{Q_p} \chi_p(ax^2 + bx)dx = \lambda_p(a)|2a|_p^{-1/2}\chi_p\left(-\frac{b^2}{4a}\right), \ \ a \neq 0, \tag{4}$$

where $\chi_p(u) = \exp(2\pi i\{u\}_p)$ is a p-adic additive character, and $\{u\}_p$ denotes the fractional part of $u \in Q_p$. $\lambda_p(\alpha)$ is an arithmetic, complex-valued function [9]. To explore the existence of a vacuum state in p-adic quantum theory we need

$$\int_{Z_p} \chi_p(ax^2 + bx)dx = \begin{cases} \Omega(|b|_p), & |a|_p \leq 1, \\ \frac{\lambda_p(a)}{|2a|_p^{1/2}}\chi_p\left(-\frac{b^2}{4a}\right)\Omega\left(\left|\frac{b}{2a}\right|_p\right), & |4a|_p > 1, \end{cases} \tag{5}$$

where Z_p is the ring of p-adic integers ($Z_p = \{x \in Q_p : |x|_p \leq 1\}$) and Ω is the characteristic function of Z_p. It should be noted that Ω is the simplest vacuum state in p-adic quantum theory.

There is quite enough similarity between real numbers and p-adics and the corresponding analysis for the so called adelic approach in mathematics [12] and physics (see e.g. Ref. 4), that in a sense unifies real and all p-adic number fields.

3 Path Integral in Ordinary and p-Adic Quantum Mechanics

According to Feynman's idea [13], quantum transition from a space-time point (x', t') to a space-time point (x'', t'') is a superposition of motions along all possible paths connecting these two points. Let us remind the corresponding probability amplitude is $\langle x'', t''|x', t'\rangle = \sum_q e^{\frac{2\pi i}{h}S[q]}$. Dynamical evolution of any quantum-mechanical system, described by a wave function $\psi(x, t)$, is given by

$$\psi(x'', t'') = \int_{Q_\infty} K(x'', t''; x', t')\psi(x', t')dx', \tag{6}$$

where $K(x'', t''; x', t')$ is a kernel of the unitary evolution operator $U(t'', t')$. In Feynman's formulation of quantum mechanics, $K(x'', t''; x', t')$ was postulated to be the path integral

$$K(x'', t''; x', t') = \int_{(x',t')}^{(x'',t'')} \exp\left(\frac{2\pi i}{h}\int_{t'}^{t''} L(q, \dot{q}, t)dt\right) Dq, \tag{7}$$

where $x'' = q(t'')$ and $x' = q(t')$.

As we know, for a classical action $\bar{S}(x'', t''; x', t')$, which is a polynomial quadratic in x'' and x', the corresponding kernel K (for one-dimensional quantum system) reads

$$K(x'', t''; x', t') = \left(\frac{i}{h}\frac{\partial^2 \bar{S}}{\partial x'' \partial x'}\right)^{\frac{1}{2}} \exp\left(\frac{2\pi i}{h}\bar{S}(x'', t''; x', t')\right). \tag{8}$$

It can be rewritten in the form more suitable for generalization (at least from the number theory point of view)

$$K_\infty(x'', t''; x', t') = \lambda_\infty\left(-\frac{1}{2h}\frac{\partial^2 \bar{S}}{\partial x'' \partial x'}\right)\left|\frac{1}{h}\frac{\partial^2 \bar{S}}{\partial x'' \partial x'}\right|_\infty^{1/2} \chi_\infty\left(-\frac{1}{h}\bar{S}\right), \tag{9}$$

where $\sqrt{ia} = \sqrt{i\text{sign}a\ |a|_\infty} = |a|_\infty^{1/2}\lambda_\infty(-a)$. In (9), $\chi_\infty(a) = \exp(-2\pi i a)$ is an additive character of the field of real numbers R. D-dimensional generalization of the transition amplitude is:

$$K_\infty(x'', t''; x', t) = \lambda_\infty\left(\det\left(-\frac{1}{2h}\frac{\partial^2 \bar{S}}{\partial x''_a \partial x'_b}\right)\right)\left|\det\left(-\frac{1}{h}\frac{\partial^2 \bar{S}}{\partial x''_a \partial x'_b}\right)\right|_\infty^{1/2}$$

$$\times\chi_\infty\left(-\frac{1}{h}\bar{S}(x'', t''; x', t')\right), \tag{10}$$

where λ_∞ is defined as

$$\lambda_\infty\left(\det\left(-\frac{1}{2h}\frac{\partial^2 \bar{S}}{\partial x''_a \partial x'_b}\right)\right) = \sqrt{\frac{1}{i^D} sign \det\left(-\frac{1}{2h}\frac{\partial^2 \bar{S}}{\partial x''_a \partial x'_b}\right)}, \tag{11}$$

and $x = (x_a), \quad a = 1, 2, \cdots, D$. By defining $\lambda_\infty(0) = 1$, one can see that this λ_∞-function satisfies the same properties as λ_p.

In p-adic quantum mechanics dynamical differential equation of the Schrödinger type does not exist and p-adic quantum dynamics is defined by the kernel $K_p(x'', t''; x', t')$ of the evolution operator:

$$\psi_p(x'', t'') = U_p(t'', t')\psi_p(x', t') = \int_{Q_p} K_p(x'', t''; x', t')\psi_p(x', t')dx'. \tag{12}$$

All general properties which hold for the kernel $K(x'',t'';x',t')$ in standard quantum mechanics also hold in p-adic case, where integration is now over Q_p. p-Adic generalization of (7) for a harmonic oscillator was done in [10] starting from

$$K_p(x'',t'';x',t') = \int_{(x',t')}^{(x'',t'')} \chi_p \left(-\frac{1}{h} \int_{t'}^{t''} L(q,\dot{q},t)dt \right) \prod_t dq(t) \qquad (13)$$

($h \in Q$ and $q,t \in Q_p$). In (13) $dq(t)$ is the Haar measure and p-adic path integral is the limit of a multiple Haar integral. This approach was extended in Ref. 14.

A rather general path integral approach, valid for analytical classical solutions, was developed for quadratic p-adic quantum systems in Ref. 15 (in one-dimension)

$$K_p(x'',t'';x',t') = \lambda_p \left(-\frac{1}{2h} \frac{\partial^2 \bar{S}}{\partial x'' \partial x'} \right) \left| \frac{1}{h} \frac{\partial^2 \bar{S}}{\partial x'' \partial x'} \right|_p^{\frac{1}{2}} \chi_p(-\frac{1}{h} \bar{S}(x'',t'';x',t')) \qquad (14)$$

and Ref. 16 (two-dimensional case).

The obtained p-adic result (14) has the same form as (11) in the real case. The higher-dimensional p-adic kernel was also considered [17]. Considering real-ordinary and all p-adic quantum mechanics on the same foot, adelic formulation is also possible [18]. It could be a good starting point to consider quantum mechanical analog of “real“, “p-adic“ and, possibly, “adelic“ tachyons.

4 p-Adic Strings and Tachyons

The p-adic **open** string theory can be deduced from ordinary bosonic open string theory on a D-brane by replacing the integral over the real worldsheet by p-adic integral [19].

A tachyon was defined as a particle that travels faster than light, and consequently has negative $mass^2$. Surely, it is not a convincing case for the tachyon. Quantum field theory offers a much better framework for considering such a pretty exotic physical model. If we would carry out perturbative quantization of the scalar field by expanding the potential around $\phi = 0$, and ignore higher (cubic, ...) terms in the action, we would find a particle-like state with $mass^2 = V''(0)$. In the case of $V''(0) < 0$ we have again a particle with negative $mass^2$, *i.e.*, a tachyon. The physical interpretation is that the potential $V(\phi)$ has a maximum at the origin and hence a small displacement of ϕ will make it grows exponentially. It is associated with the instability of the system and a breakdown of the theory.

Conventional formulation of string theory uses a first quantized formalism. In this formulation one can get a state-particle with negative $mass^2$,

i.e. tachyons. The simplest case appears in 26 dimensional bosonic string theory. This approach is, unfortunately, not suitable for testing tachyon‘s solutions [2], but there are superstring theories defined in (9+1) dimensions that have tachyon free closed string spectrum. In addition, some string theories contain open string excitations with appropriate boundary conditions at the two ends of the string. So, one can ask: is there a stable minimum of the tachyon potential around which it is possible to quantize the theory. In the last few years there many papers devoted to this problem have made some progress, but we will not consider them in this paper.

In p-adic string theory all tree level amplitudes involving tachyons in the external states can be computed. The p-adic (open) string theory is obtained from ordinary bosonic (open) string theory on a D-brane by replacing the integral [19] over the real world-sheet coordinates by p-adic integral associated with a prime number p. There have been somewhat different approaches, but we will not consider the constructions of all these theories.

Let us see the exact effective action for the p-tachyon field. It is described by the lagrangian [21]

$$L_p = -\frac{1}{2}\phi p^{-1/2\Box}\phi + \frac{1}{p+1}\phi^{p+1}. \tag{15}$$

This form, obtained by computing Koba-Nielsen amplitudes for a prime p, makes sense for all (*integer*) values of p. The classical equation of motion derived from (15) is

$$p^{-1/2\Box}\phi = \phi^p. \tag{16}$$

Besides the trivial constant solutions $\phi = 0, 1$, a soliton solution is admitted. The equations separate in the arguments and for any spatial direction x we get

$$\phi(x) = p^{1/2(p-1)} \exp\left(-\frac{p-1}{2p\ln p}x^2\right), \tag{17}$$

a gaussian lump whose amplitude and spread are correlated [5].

5 Quantum Mechanical Analogue of Tachyon Matter

Now, we will concentrate on a relatively new field theory - the field theory of tachyon matter was proposed by Sen a few years ago [20]. The derivation of its action is based on a rather involved argument. The obtained form is pretty strange and different from the actions we used to be familiar ones

$$S = -\int d^{n+1}x V(T)\sqrt{1+\eta^{ij}\partial_i T\partial_j T}. \tag{18}$$

Let us consider p-adic analogue of the above action, originally considered as real one, *i.e.* $\eta_{00} = -1$, $\eta_{\mu\nu} = \delta_{\mu\nu}$, where $\mu, \nu = 1, 2, ..., n$. $T(x)$ is a p-adic scalar tachyon field and $V(T)$ is the tachyon potential: $V(T) = \exp(-\alpha T/2)$. In the bosonic case, $n = 25$, $\alpha = 1$, and for superstring $n = 9$, $\alpha = \sqrt{2}$. The square root appearing in action (18) (and its multiplication to tachyon potential) makes this theory so unusual.

Here we examine a lower (zero-dimensional) mechanical analogue of the field theory of p-adic tachyon matter (whatever it would physically mean). As usually, the correspondence can be obtained by the correspondence $x^i \to t$, $T \to x$, $V(T) \to V(x)$. The corresponding zero-dimensional action reads

$$S_0 = -\int dt V(x)\sqrt{1 - \dot{x}^2}, \tag{19}$$

where integration has to be performed over p-adic time. From the above action it is not difficult to get the classical equation of motion

$$\ddot{x} + f(x)\dot{x}^2 = f(x), \tag{20}$$

where function $f(x)$ denotes

$$f(x) = -\frac{1}{V}\frac{\partial V}{\partial x}.$$

Partial differentiation of p-adic valued function is well defined, although in this case it can be replaced by the ordinary one, because $V = V(x)$.

Keeping in mind that exponential p-adic function (the tachyon potential $V(x) = \exp(-\alpha x)$ should be understood as an analytic function with corresponding radius of convergence [9] $r \sim 1/p$, we obtain as in the real case

$$\ddot{x} + \alpha\dot{x}^2 = \alpha. \tag{21}$$

By the replacements $\dot{x} = \gamma\dot{y}$, $\alpha\gamma = \frac{\beta}{m}$ and $\frac{\alpha}{\gamma} = g$ we get the equation of motion

$$m\ddot{y} + \beta\dot{y}^2 = mg, \tag{22}$$

which describes motion of a particle of mass m moving in a constant (say gravitational, Newtonian) field with quadratic friction. It is interesting that this equation can be derived from the (p-adic) action

$$S_0^{(p)} = -\int dt e^{-\frac{\beta y}{m}}\sqrt{1 - \frac{\beta}{mg}\dot{y}^2}. \tag{23}$$

Surprisingly or not, the zero dimensional analog of the (Sen's) field theory of tachyon matter offers an action integral formulation for the system under gravity in the presence of (quadratic) damping. The solution of the equation of motion (22) reads

$$y = y_0 + \frac{m}{2\beta} \ln \left(\frac{g - \frac{\beta}{m} v_0^2}{g - \frac{\beta}{m} v^2} \right), \tag{24}$$

with initial $t = 0$ conditions for position $y(0) = y_0$ and velocity $v(0) = v_0$. This solution has the same form in the real and p-adic case, but the radius of convergence is rather different.

Faced with the increasing interest in various aspects of tachyon field theory, including its p-adic aspect, this connection with the field theory of tachyons through action integral formulation seems worth mentioning and examining in general. Also, quantization of the theory in path integral language might be very useful and, as we know, very general (for real, p-adic and adelic path integrals see, e.g. Ref. 17). However, a kernel of the operator of evolution that corresponds to the action (23) is still unknown, even in the real case. Because of that the square root and exponential for small β should be expanded. If we treat β p-adicaly small, in respect to p-adic norm, we obtain

$$S_0 \sim - \int dt \left(1 - \frac{\beta y}{m} - \frac{\beta}{2mg} \dot{y}^2 \right). \tag{25}$$

We have already calculated the path integral for the particle in constant external field [14]. Here we have a slightly changed form ($y'' = y(\tau),\ y' = y(0),\ h = 1$)

$$K_p(y'', \tau; y', 0) = \lambda_p \left(\frac{\beta}{mg\tau} \right) \left| \frac{mg\tau}{2\beta} \right|_p^{-\frac{1}{2}}$$
$$\times \chi_p \left[- \frac{\beta (y'' - y')^2}{mg\tau} - (\frac{\beta (y'' + y')}{2m} - 1) \tau + \frac{1}{48} \frac{\beta}{m} g \tau^3 \right]. \tag{26}$$

It makes it possible to check the existence of the simplest tachyonic vacuum state (invariant in respect to the evolution operator), of the corresponding quantum mechanical model, *i.e.*

$$\Omega(|y''|_p) = \int_{|y'| \leq 1} K_p(y'', \tau; y', 0) \Omega(|y'|_p) dy'. \tag{27}$$

Using (5) and (26) we find that for the existence of the "ground" (Ω) state of (quantum) p-adic tachyons (here some technical details and case p=2 are omitted) the following is necessary $\left| \frac{\beta}{m} \left(\frac{2y''}{g\tau - \frac{\tau}{2}} \right) \right|_p \leq 1$ for $\left| \frac{\beta}{mg\tau} \right| \leq 1$, *or* $\left| 2y'' - \frac{g\tau^2}{2} \right|_p \leq 1$ for $\left| \frac{\beta}{mg\tau} \right|_p > 1$. Possible physical implication on constraints for quantities related to the starting tachyon action (18) will be discussed elsewhere. The existence of Ω state opens the "door" for further adelic generalization and investigation of higher-excited states.

As in the real case [8] a quadratic damping effect could enter explicitly into the play treating it as a perturbation over classical solution of the equation for

a particle in constant external field without friction. Damping effect would be ascertained and understood through its dependence of β term [22] .

6 Conclusion

In this paper we show that quantum mechanical simplification of the tachyon field theory, besides the real case, is possible in a p-adic context. Also, an adelic generalization looks possible, *i.e.* without some obvious principal obstacles. Path integral formulation of zero-dimensional p-adic tachyons has been done and some "minimal" conditions for their existence have been found. Of course, how much this approach could be useful for deeper understanding of the whole string theory and of its tachyon sector requires time and further, in-depth research. The fact, that the exact effective tachyon action in the usual string theory is not known, while in p-adic string theory it is, is quite enough motivation for this and similar investigations.

We would propose a few promising lines for further investigation. The exact formula for quadratic quantum p-adic systems in two and more dimensions [17] could be useful for multidimensional generalization of p-adic tachyons. It is tempting to extend our approach to $1 + 1$ dimensional field theories, even nonlinear field theories would be here quite nontrivial problem.

Finally, p-adic string theory could be a very useful guide to difficult question in the usual string theory. It requires deeper understanding of p-adic string theory itself, especially of closed p-adic strings (strings on p-adic valued worldsheet and target space as well). It is a worthwhile task to explore p-adic strings in nontrivial backgrounds. It will naturally lead to *noncommutative* formulation on p-adic quantum theory and examination of the corresponding Moyal-product, introduced in a context of the noncommutative adelic quantum mechanics [23]. Recently, the Moyal-product has been applied in the calculation of the noncommutative solitons in p-adic string theory [24].

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